

[4]

BCA-105(N)

Section—C

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

6. Determine the eigen value and the eigen vector of the matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

7. Differentiate the following function w.r.t. x and find n th derivative :

(i) $\cos x \cos 2x \cos 3x$

(ii) $e^{2x} \cos^2 x \cdot \sin x$

(iii) $\frac{1}{1+x+x^2+x^3}$

8. (a) Show that the function defined by :

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

is continuous at $x = 3$.

- (b) Find the values of a and b such that :

$$\lim_{x \rightarrow 0} \frac{x(a + b \cos x) - c \sin x}{x^5} = 1$$

9. Express :

$$\begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{vmatrix}$$

as the square of a determinant and hence find its value.

BCA-105(N)

4,000

(C-95)

Roll No.

BCA-105(N)

B. C. A. (First Semester)

EXAMINATION, Dec., 2019

(New Course)

Paper Fifth

MATHEMATICS—I

Time : Three Hours]

[Maximum Marks : 75

Note : Attempt questions from all Sections as directed.

Inst. : The candidates are required to answer only in serial order. If there are many parts of a question, answer them in continuation.

Section—A

(Short Answer Type Questions)

Note : Attempt all questions. Each question carries 3 marks.

1. (A) Using Cramer's rule, solve the following system of equation :

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

(C-95) P. T. O.

[2]

BCA-105(N)

(B) Find the value of :

$$\int \frac{\sin^4 x}{\cos^2 x} dx$$

(C) Evaluate the following :

$$(i) \lim_{x \rightarrow 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\log x}{x}$$

(D) Find C of the mean value theorem, when :

$$f(x) = x^3 - 3x - 2$$

in $[-2, 3]$.

(E) Find the rank of the matrix :

$$A = \begin{bmatrix} 5 & 6 & 7 & 8 \\ 6 & 7 & 8 & 9 \\ 11 & 12 & 13 & 14 \\ 16 & 17 & 18 & 19 \end{bmatrix}$$

(F) Find the maximum and minimum value of the function :

$$f(x) = x^3 - 3x^2 + 6$$

(G) Evaluate :

$$\int_0^1 x^4 (1 - \sqrt{x})^5 dx$$

(H) If:

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

$$\text{and } \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$$

find the value of :

$$(\vec{a} + 4\vec{b}) \cdot (2\vec{a} - \vec{b})$$

(C-95)

[3]

BCA-105(N)

(I) Differentiate the following function w.r.t. x :

$$(i) \frac{\sin x - x \cos x}{x \sin x + \cos x}$$

$$(ii) (\log x)^{\sin x}$$

Section—B

(Long Answer Type Questions)

Note : Attempt any *two* questions. Each question carries 12 marks.

2. Show that the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

satisfies its own characteristic equation, find A^{-1} .

3. Prove that the area of the triangle whose vertices are $\vec{A}, \vec{B}, \vec{C}$ is :

$$\frac{1}{2} \left[\vec{B} \times \vec{C} + \vec{C} \times \vec{A} + \vec{A} \times \vec{B} \right]$$

4. If: $y = \left[x + \sqrt{x^2 + 1} \right]^m$

prove that :

$$(i) (x^2 + 1)y_2 + xy_1 - m^2 y = 0$$

$$(ii) y_{n+2} + (n^2 - m^2)y_n = 0 \text{ at } x = 0$$

5. By using Maclaurin's theorem, find the first four terms in the expansion of :

$$\log(1 + \tan x)$$

in the power of x .

(C-95) P. T. O.